Dominoes and beyond—Teachers Circle April 2009

Tiling problems. In typical tiling problems we are usually concerned with covering a region (often a large rectangle) with one or more kinds of smaller tiles, possibly of various shapes. There are three ground rules:

a. The region must be *entirely* covered.

b. No two of the tiles may overlap

c. No part of any tile may lie outside of the region.

If all three of these conditions are satisfied, we say that the region can be tiled by the set of tiles. A couple of questions come to mind:

Given a region and a set of tiles, can the tiles be used to cover the region?

If a region can be tiled by a set of tiles, in how many different ways can this be done. (And by the way, what do you mean by "different"?)

1. We are given an 8×8 checkerboard and a collection of 2×1 tiles (e.g.dominoes). Each domino can be placed to cover exactly two (adjacent) squares of the checkboard.

   a. If we remove two opposite corner squares from the checkerboard can the remaining board be tiled with the dominoes?

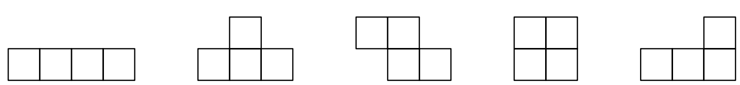
   b. Suppose we can remove any two squares from the checkerboard? For which choice of removed squares can the board be tiled with dominoes?

2. We again have our 8×8 checkerboard but now have a collection of 3×1 tiles (triomiinoes.)

   a. If we remove one corner square from the checkerboard can the remaining board of 63 squares be tiled with the triominoes?

   b. Suppose we can remove any one square from the checkerboard. For which choices can the resulting board be tiled with trinominoes?

3. The five tetraminoes are pictured below. Can these five pieces by assembled to form a rectangle?

   From left to right the tetraminoes are named *straight, T, Z, square, L.*

4. Can an 8×8 checkerboard be tiled with 16 T tetranimoes? Can a 10×10 board be tiled with 25 T tetraminoes?

5. A rectangular checkerboard is tiled with *square* and *straight* tetraminoes. One of the square tiles gets broken and you only have one extra straight tile to replace it. Can the tiles be rearranged so the board is again tiled?

6. You have a collection of 1×1 and 1×2 tiles. In how many different ways can a 1×12 board be covered with such tiles? (Two tilings are different if one tiling has a different number of tiles of each kind than the other, of if they use the same numbers of each kind of tile but in different orders.)

7. In how many ways can a 1×30 board be tiled with 1×4 and 1×5 tiles?

Domino Sets.

For positive integer n an $[n, n]$ domino set contains one domino with k dots on one half and j dots on the other for each choice of integers $0 \leq k, j \leq n$. For example, a [6, 6] set has a domino with 3 dots on one half and 4 on the other, a domino with 2 dots on each half, a domino with 0 dots on one half and 6 on the other, etc.

8. How many dominoes are in a $[1, 1]$ set? In a $[2, 2]$ set? A $[4, 4]$ set? How many are in an $[n, n]$ set?

9. A domino train for an $[n, n]$ set is a line that includes all dominoes in the set laid end to end, and such that two dominoes are adjacent only if the numbers on the two adjacent halves are the same. For which n can the dominoes in an $[n, n]$ set form a train?

10. A circular train is a domino train such that the entire domino set dominoes is laid in a circle and two abutting dominoes have the same numbers on the touching halves. For which n can the set of all dominoes in an $[n, n]$ set be used to form a circular train?