Developing Precalculus Level Students’ Mathematical Meanings and Practices: The Role of Curriculum, Teachers, and Instruction

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Passing rates in first-semester calculus by students who had passed the prerequisite precalculus ranged from 17%-67% at six colleges and universities (Schattschneider’s, 2006).
Research on calculus learning revealed that students’ weak understandings of ideas of variable, function and rate of change are obstacles for learning and understanding key ideas of calculus.

- **Limit** (Cottrill, Brown, Dubinsky, 1998; Jacobs, 1999; Oehrtman, 2009; Williams, 2002)

- **Derivative** (Tall, 1992; Zandieh, 2004)

- **Accumulation** (Carlson, Oehrtman, & Thompson 2009; Smith, 2009; Carlson, Larsen, Jacobs; 2007)

- **Related Rate Problems** (Engelke, 2008; Carlson, 2007)

- **Fundamental Theorem** (Thompson, 1994; Carlson, Oehrtman, Thompson, 2009; Smith, 2009)
Possible Sources of the Problem

- **Students are not constructing the foundational understandings and reasoning abilities that are necessary for learning calculus** *(Making the Connection: Research and Teaching in Undergraduate Mathematics Education, Carlson & Rasmussen, 2009; Kaput, 1992; Dubinsky 2002; Monk, 1992; Thompson 1994; Carlson, 1998; Jacobs, 2000; Carlson, Oehrtman, Engelke, 2010)*

- **Teachers are not being supported in teaching for meaning.**
  - Most teachers rely on their textbooks and do not have the time or expertise to develop high quality curriculum materials *(Oehrtman, 2009; Teuscher, 2010; Thompson, 2008)*
What reasoning abilities, major ideas, procedures and mathematical practices do students need to learn in a precalculus level course to prepare students for success in calculus?

What experiences and instruction will support student development of these ideas and abilities?
What shifts are needed in teachers to support students in developing foundational understandings, reasoning abilities and mathematical practices/problem solving abilities called for in the CCMS?

What support do teachers need to make these shifts?
What understandings of these foundational precalculus ideas are common in students?

- **Variable:**
  - Something to “solve for” (Schoenfeld, 1989; Jacobs, 1999)

- **Function:**
  - Two expressions separated by an equal sign (Dubinsky, 1992; Thompson, 1994)

- **Average Rate of Change:**
  - Add them up and divide

- **Proportionality:**
  - Cross multiply
Exponential function

\[ f(x) = x^2 \]

Given \( f \) is defined by \( f(x) = 2x^2 + 3x \)

What is \( f(x+a) \)?

\[ f(x + a) = 2x^2 + 3x + a \]

\( f(g(x)) \) means \( f \) times \( g \)

The phrase “write a function that expresses \( s \) in terms of \( t \)” means:

there must be an \( s \) and a \( t \) and an equal sign
A ball is thrown into a lake, creating a circular ripple that travels outward at a speed of 5 cm per second. Express the area, $A$, of the circle in terms of the time $t$ in seconds that have passed since the ball hits the lake.
A ball is thrown into a lake, creating a circular ripple that travels outward at a speed of 5 cm per second. Express the area $A$ of the circle in terms of the time $t$ in seconds that have passed since the ball hits the water.

- a) $A = 25\pi t$ (21%)
- b) $A = \pi r^2$ (7%)
- c) $A = 25\pi t^2$ (17%)
- d) $A = 5\pi t^2$ (34%)
- e) None of the above (20%)

WHY ARE WORD PROBLEMS SO DIFFICULT FOR STUDENTS?
What are the dimensions of a box with the maximum volume that can be formed from cutting equal squares from the corner of an 11” X 13” piece of paper?

Occurred after he had created a diagram and determined $V=13 \times 11 \times x$

- Travis appeared to focus on the object of the box, but not the attributes of length and width of the box
  - The length and the width of the box had fixed measurements. The various references to length were not distinct in Travis’ mind.
Travis appeared to conceive of the various quantities (attributes of the box that could be measured) and how they change together.

Tavis defined and conceptualized variables and then constructed a formula (for the box’s volume in terms of the cutout length) that was meaningful to Travis.
Foundational Understandings and Reasoning Abilities for Calculus

- Proportion, Constant Rate of Change, The Meaning of Ave ROC, Modeling Linear growth
- Function (Composition and Inverse)
- Exponential Functions
- Polynomial Functions, Changing Rate of
- Rational Functions
- Trigonometric Functions
  - Unit Circle, then triangle trig.
One Model for Supporting Teachers and Students
Developing the Pathways to Calculus Professional Development Model

Review research to provide the theoretical grounding

--Cognitive Models of Learning Key Ideas
-Models of How Mathematical Practices are Acquired
--Models for Effective Learning Communities
-Models of how MKTP Develops

Design
Implementation (Revise)

-Implement
-Study Implementation

Revise
Frameworks
Primary Phase I Findings

- Improving teachers’ understanding of key ideas of courses they teach is not sufficient to improve teachers’ instruction.

- PLC’s can be highly productive in supporting teachers to examine student thinking if they have a strong facilitator who is coached by project leaders (e.g., Carlson, Moore, Bowling; 2007).

- Mandated assessments (state and district exams) are obstacles to changing teaching practices.

- School culture (unsupportive administrators, focus on developing procedural knowledge) is difficult to change.
Adjusted Intervention: Pathways Adopted Schools

- Administrators supportive
- All teachers were required to participate

Our research continued to reveal that we were having little impact on teaching and student learning

- Teachers were generally not effective in modifying their curriculum or teaching practices to focus more on student understanding and student thinking.
  - Teachers’ new understandings were fragile (had not yet become spontaneous when speaking, lacked important connections)
- Teachers were unable to act effectively in the moment of teaching to support student learning
We continued our work to support the development of teachers MKT

- Understandings that allow teachers to spontaneously act in ways to teach effectively
The Pathways Professional Development Model

- Must include
  - student curriculum that is based on theory of how students learn
  - Instructional supports to advance teachers’ key developmental understandings and theories of how students learn specific ideas

The Pathway Precalculus Curriculum
Pathways Phase II is scaling (and studying the scaling) of the *Pathways Professional Development Model for Teaching Secondary Mathematics*

- **Workshops/graduate courses** focused on developing mathematical content knowledge for teaching key ideas and reasoning abilities of algebra through precalculus

- **PLC’s** ("Effective PLC’s (FOP) (Carlson, Moore, et al., 2007)"

- **Professional support tools** for teachers (student level tasks, embedded research knowledge, conceptually oriented assessments.) (Carlson, Oehrtman, Moore, Strom)
Laying the Foundation for Success in Calculus (and Beyond)

- Tasks support students in developing foundational reasoning abilities and understandings.

- Classroom tasks engage students in meaning making and promote the development of students’ problem solving abilities.

- Teacher Support Tools Help Teachers Acquire “Content Knowledge for Teaching”
  - PowerPoint Slides with Dynamic Animations and Links to Applets Help Novice Teachers Engage Students in Conceptual Conversations.
  - Worksheets (with detailed solutions) help teachers promote critical reasoning abilities and understandings in students.

See PD Tools.
Classroom Environment that Promotes Expert Mathematical Practices:

Individuals are expected to:

- Persist in sense making
  - Attempt to make logical connections
- Exhibit Mathematical Integrity
  - Don’t pretend to understand when you don’t
  - Base conjectures on a logical foundation
- Students are expected to express their thinking and *speak with meaning* about their problem solutions
- Students are expected to listen to the *meaning* being conveyed by their peers and ask questions when the *meaning* is not clear
The Precalculus Concept Assessment Instrument

- A 25 item multiple choice exam
- Based on the body of literature related to knowing and learning the concept of function (Carlson, 1998, 1999, 2003; Dubinsky et al., 1992; Monk, 1992; Thompson and Thompson, 1992; Thompson, 1994; Sierpinska, 1992)
- Validated and refined over a 12 year period
Student PCA Performance

- PCA administered to 550 college algebra and 379 pre-calculus students at a large southwestern university.
- Also administered to 267 pre-calculus students at a nearby community college.

- Mean score for college algebra: 6.8/25
- Mean score for pre-calculus: 9.1/25
Predictive Potential of PCA

- How does PCA score relate to course grade in calculus?
  - 83% of 277 students who scored 13 or above on PCA at the beginning of the semester received an A, B, or C in calculus.
  - 85% of the 277 students who scores 11 or below received a D, F or withdrew.
## Summary of PCA pre- and post-test

### College Algebra (ASU)

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<th>Pre-test Mean</th>
<th>Post-test Mean</th>
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<tbody>
<tr>
<td>Fall 11</td>
<td>5.7</td>
<td>12.2</td>
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(Previous Best Mean Score: 6.8)

### Precalculus (Secondary Schools)

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<th>Pre-test Mean</th>
<th>Post-test Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall 11</td>
<td>8.2</td>
<td>16.2</td>
</tr>
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(Previous Best Mean Score: 910.1)
For more information about piloting Pathways Precalculus contact:

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Subject Line: Iowa State Precalc Conf
Interested in PCA
or Interested in piloting Pathways to Calculus curriculum materials