Comparing Similar Triangles and Beginning Trigonometric Ratios.

In this activity the students will gather data from our football field to calculate the height of objects that they cannot measure directly. The students will use two or three methods to calculate the height of the objects depending on the amount of time we have available. Then the students will compare the calculated heights to see how close the three of them are to each other and to their fellow classmates.

I usually put the students into groups of three: one student will record the data, one student will be responsible for taking measurements, and the third will be responsible to sight the objects. I have the students keep the same job for the entire time, but the students could also switch jobs for each of the three methods of measurement. This is a three or four day activity and works well during homecoming week if it falls late enough in the school year. The materials needed are meter sticks, long measuring tapes (100 meter), the student made hypsometers, the student made clinometers, and the sun.

I start the activity with discussions about what will be happening and the students responsibilities, possible places for errors, ways to get better results, possible applications. Once the measurements are done and the calculations are complete we compare results, discuss possible points of error, and discuss applications of each method.

The first method is to use shadows. The students measure the length of the shadow of the object and the length of the shadow of a meter stick. Then they are able to use similar triangles to solve for the height of the object. One common place for errors with this method is for students to measure the shadow of the meter stick only once and not remeasure for each object. As the sun moves during the class period it can cause large errors in the calculations. It is something we discuss during the start of the project, but not something I explicitly remind them about during the data gathering. This often brings some interesting conversations between and within the groups. Then I have the students use the Pythagorean Theorem and trigonometry to solve for the missing parts of the two triangles and compare the similar triangles to see if they came up with the logical values.

The second method is to use similar triangles and a hypsometer. The students build the hypsometers in class. It consists of a stiff piece of cardboard (solid cardboard like from the back of a legal pad works better than corrugated), a ruler and a weight on the string. The students punch a hole in the cardboard to hang the weight on the string through. Then they secure the ruler to the bottom edge of the cardboard so that the zero lines up the weight hanging straight down. (I bring in a level for them to be sure that it is aligned properly.) The students measure the distance to the object, the height of the sighters eye, and where the string crosses the ruler on the bottom of the hypsometer. Then making the similar triangles they are able to calculate the height of the object. A common error occurs with finding which sides of the hypsometer align with which sides of the sighted object. Again, I have the students use the Pythagorean Theorem and trigonometry to solve for the missing parts of the two triangles and compare the similar triangles to see if they came up with the logical values. This also helps with being sure that they have lined them up correctly.
The third method is to use a clinometer (sextant). The students build the clinometer in class. It consists of a protractor and a weight on the string. The students put the weight on the string through the hole at the middle of the protractor. The students measure the distance to the object and the angle on the clinometer. Then they are able to use similar triangles and trigonometry to solve for the object’s height. The students solve for the missing part of the triangle with trigonometry.

The students then compare the three heights of the object to see how they compare. We also compare them as a class to see if any of the groups are significantly different than the rest of the class. We discuss places for errors, advantages and disadvantages for each method, and times or places where certain methods work or do not work.

The students seem to enjoy the activity and opportunity to be outside and find applications for the mathematics that we are studying. I get a large variety of answers, but most often they line up pretty well and we as a class can make a logical guess as to the actual height of the objects. I do not know or use the actual height of the objects because I think the discovery and application is more important than getting it to match a result. Also, the argument could be made that if I know the height why do all the work to find it again. I have the groups go to different objects to measure so that there isn’t a traffic jam at any single object.

I like this activity because of the mathematical discussions I see the students involved in during the activity. The logic that plays out is interesting as well. Some groups will stay in one spot to gather all the data for all three methods, while others change for each one. I generally get pretty good results with only one or two groups having answers that are not reasonable.

I grade the students equally on four areas for the activity. The four areas are their participation, the calculations they make, the comparison of the three methods, and results they find. The participation is separate for each student and is a combined score given by me and the other members of the group. The other three areas are shared for the group because I have one packet for each group rather than one for each student.