

For positive integers  $m, n$  define

$$a(m, n) = j \quad \text{where} \quad 0 \leq j \leq 6 \quad \text{and} \quad n^m \equiv j \pmod{7}.$$

For  $i \in \{0, 1, 2, 3, 4, 5, 6\}$ , and positive integer  $k$ , let

$$f(i, k) = \text{the number of elements in the set} \\ \{(m, n) : a(m, n) = i \quad \text{and} \quad 1 \leq m, n \leq k\}.$$

Prove that

$$\lim_{k \rightarrow \infty} \frac{f(i, k)}{k^2}$$

exists for each  $i \in \{0, 1, 2, 3, 4, 5, 6\}$ .