

Let $x_i, y_i, 1 \leq i \leq 3$ be real numbers with

$$x_1 \leq x_2 \leq x_3 \quad \text{and} \quad y_1 \leq y_2 \leq y_3,$$

and let α, β, γ be nonnegative real numbers with $\alpha + \beta + \gamma = 1$. Prove that

$$\begin{aligned} &\alpha x_1 y_1 + \beta x_2 y_2 + \gamma x_3 y_3 \\ &\geq (\alpha x_1 + \beta x_2 + \gamma x_3)(\alpha y_1 + \beta y_2 + \gamma y_3). \end{aligned}$$

This problem was posed by Prof. Krishna Athreya.