

A positive integer is called *square free* if it is not divisible by the square of any prime number. The first few square free positive integers are

1, 2, 3, 5, 6, 7, 10, 11, 13, 14, 15, 17,

At Infinity High School there are infinitely many lockers and infinitely many students. The lockers are all on one wall of a long wall and numbered, in order, 1, 2, 3, . . . , and initially all are closed. Each student in the school is assigned one square free number, so the students are numbered 1, 2, 3, 5, 6, 7, 10, 11 The math teacher has the students perform an experiment. One at a time he sends the students down the hall (in order) with these instructions: “if your number is k , then starting with locker k you are to change the state of every k th locker: open it if it is closed and close it if it is open. Do not touch any other lockers.” (So student 1 would open every locker since all are initially closed; student 2 would then close lockers 2, 4, 6, . . . , and so on.)

When all students have gone down the hall and followed the teacher’s instructions, which lockers will be open? Prove that your answer is correct.