The positive integers 1, 2, 3, . . . , 1000 are written on a blackboard. The following process is then performed repeatedly:

> Pick at random two numbers from the board, say u and v. Erase u and v and then write on the board the number $uv + u + v$.

Thus, with each reptition of the above, the number of numbers on the board desceases by one. Continue this process until there is just one number left on the board. Describe completely the set of possibities for this last number, and prove that your result is correct.