Let S be a nonempty set with at least three elements, and let $f : S \to S$ be a one-to-one onto mapping from S onto itself. Suppose that for each function $g : S \to S$ that is also one-to-one and onto, we have

$$f\left(g(x)\right) = g\left(f(x)\right).$$

Prove that $f(x) = x$ for each $x \in S$.