

For positive integer n , let $d(n)$ be the number of digits in the decimal (e.g. base 10) representation of n . For example, $d(3) = 1$ and $d(909) = 3$. Find the value of the sum

$$\sum_{n=1}^{\infty} \frac{1}{d(n)!}.$$