

A buyer and a seller bargain on the price of a GIZMO that the buyer would like to purchase and the seller would like to unload. Initially the buyer offers to pay 0 and the seller counters by asking for 1. Next the buyer offers  $2/3$ , which is his original offer plus  $2/3$  of the difference between the buyer's and seller's price. Next the seller offers  $8/9$ , dropping his price by  $1/3$  of the difference between the seller's price and the buyer's most recent offer. The bargaining continues in this way, with the two parties alternating offers. When it is the buyer's turn, he raises his latest offer by two-thirds of the difference between the most recent buyer's and seller's offers. Then the buyer decreases his price by one-third of the difference between the two latest offers.

Continuing this process we generate two sequences; a sequence of buyer's offers,

$$0 = b_0, \frac{2}{3} = b_1, b_2, b_3, \dots$$

and a sequence of seller's offers,

$$1 = s_0, \frac{8}{9} = s_1, s_2, s_3, \dots$$

Prove that both of these sequence converge and that they converge to the same number. What is this ultimate selling price (e.g. limit)?