Let S be the value of the sum. Then

$$\begin{aligned} S &= \frac{F_0}{10^0} + \frac{F_1}{10^1} + \sum_{n=2}^{\infty} \frac{F_n}{10^n} = \frac{1}{10} + \sum_{n=2}^{\infty} \frac{F_{n-1} + F_{n-2}}{10^n} \\ &= \frac{1}{10} + \sum_{n=1}^{\infty} \frac{F_n}{10^{n+1}} + \sum_{n=0}^{\infty} \frac{F_n}{10^{n+2}} \\ &= \frac{1}{10} + \frac{1}{10}\left(\sum_{n=0}^{\infty} \frac{F_n}{10^n} - \frac{F_0}{10^0}\right) + \frac{1}{10^2}\left(\sum_{n=0}^{\infty} \frac{F_n}{10^{n+2}}\right) \\ &= \frac{1}{10} + \frac{1}{10}S + \frac{1}{100}S. \end{aligned}$$

Solving for S we have $S = \frac{10}{89}$.

Some solvers used the Binet form for the Fibonacci numbers,

$$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right).$$

Substituting this for F_n in the given sum converts the problem to a sum of two geometric series.

By similar arguments, it can be shown that for $|x| < \frac{1+\sqrt{5}}{2}$,

$$\sum_{n=0}^{\infty} F_n x^n = \frac{x}{1 - x - x^2}.$$

This latter expression is the *generating function* for the Fibonacci sequence.