

Let the value of the limit be  $L$ . Then

$$\begin{aligned}\log L &= \lim_{n \rightarrow \infty} \left( -\frac{n+1}{2n} \log n + \sum_{k=1}^n \frac{k}{n^2} \log k \right) \\ &= \lim_{n \rightarrow \infty} \left( -\frac{1}{n^2} \sum_{k=1}^n k \log n + \sum_{k=1}^n \frac{k}{n^2} \log k \right) \\ &= \lim_{n \rightarrow \infty} \left( \sum_{k=1}^n \left( \frac{k}{n} \log \frac{k}{n} \right) \frac{1}{n} \right) \\ &= \int_0^1 x \log x \, dx = -\frac{1}{4}\end{aligned}$$

Therefore,

$$L = \frac{1}{\sqrt[4]{e}}.$$