

Suppose there are six cities A_1, A_2, \dots, A_6 connected by direct flights to a city P . Then there is a pair i, j so that the measure of (acute) $\angle A_i P A_j$ is at most 60° . Without loss of generality let $\angle A_1 P A_2$ have measure less than 60° . Because the distance between each pair of towns is different than the distance between any other pair, $\triangle A_1 P A_2$ is scalene, and the largest angle has measure greater than 60° . We may assume, again without loss of generality, that this is $\angle A_2 A_1 P$. Thus

$$PA_2 > PA_1 \quad \text{and} \quad A_2 P > A_2 A_1.$$

Thus A_2 is not the city closest to P , and P is not the city closest to A_2 . This contradicts the assumption that there is a direct flight between P and A_2 .

