Automated discharging arguments for density problems in grids

Motivated by problems in wireless sensor networks, we consider minimizing the density of an identifying code in the hexagonal grid. An identifying code is a set of vertices where every vertex in the grid is uniquely identified by its adjacent code elements. While the minimum density of an identifying code is known for the square and triangular grids, there is still a gap between the upper and lower bounds for the hexagonal grid. Most lower bounds are found using discharging, which is a method to demonstrate the interaction between local structure and global averages.

Verifying a discharging proof is straightforward but usually very tedious. However, the creation of a discharging argument can be very mysterious. We will present a new computer-automated approach to not only verify discharging arguments, but also to generate them from scratch. A critical component to this method is solving a linear program that will assign value to the specified discharging rules, resulting in the best possible proof using those rules. Using this method, we find a new lower bound of $\frac{23}{55}$ (approximately 0.4181818) on the density of an identifying code in the hexagonal grid, improving on the current-best human-created proofs.